

LESSON PLAN TEMPLATE

Teacher: _____

Grade level: _____

Title of Lesson: Infinite Series

Subject/Course: Algebra

Date(s) presented: TBD

Lesson duration: 1-2 hours

STAGE 1 – Desired Results

Established Goals:

- What relevant goals (content standards, course or program objectives, learning outcomes, academic language goals, etc.) will this lesson address?
 - What **academic standards** will this lesson address?
 - NCTM Algebra Standard: "Generalize patterns using explicitly defined and recursively defined functions"
 - NCTM Algebra Standard: "Use symbolic algebra to represent and explain mathematical relationships"
 - **Socio-mathematical Goals:**
Which of the following goal(s) will receive attention during your lesson?
 - Students will justify their work both in terms of why they selected a particular approach and reasons their answers are reasonable.
 - Students will ask each other questions that press for reasoning, justification, and understanding.
 - Students will compare strategies and results in order to learn from each other's work.
 - Students will use mistakes as a site for new learning.

Understandings:

Students will understand that...

- Infinite series can have finite sums
- Patterns in infinite series and how an infinite series can have a finite sum
- An infinite series does not necessarily add up to infinity

Essential Questions:

- From your model and diagram, what will $1/2 + 1/4 + 1/8 + 1/16 + 1/32 + 1/64 + \dots$ equal?
- What fraction do these two pieces represent?
- What real life scenario can you think of where you will need to solve an infinite series?

Students will know...

- How to approach the sum of an infinite series through modeling
- About the patterns in infinite series and how to determine the value of each term in a series

Students will be able to...

- Use symbolic algebra to represent the sum of infinite series and situations which require the use of infinite series

STAGE 2 – Assessment Evidence

Performance Tasks:

- Informal observations of students' written responses to the teacher's prompts in their notebooks as well as their comments during class discussion

Other Evidence:

- Homework Assignment – Students ability to solve the series and their explanation of their strategy

STAGE 3 – Learning Plan

Learning Activities:

- Ask students to fold the blank sheet of paper in half and cut it into two rectangles. Remind students that the sum they have in front of them is " $1/2 + 1/2 = 1.$ " **H**
- Ask students how they can represent $1/2 + 1/4$ using the pieces in front of them. If students do not figure it out on their own after a couple of minutes, instruct them to put aside one of the half rectangles and to take the other half, fold that one in half, and cut it into two squares, thus creating a piece that is

one-fourth of the original blank sheet. **W, E**

- Before students continue the pattern, ask students to hypothesize what they think will happen if they continue this pattern and to write down their ideas in their notebooks. **R**
- After students finish their hypothesis, ask them to cut one of the fourth pieces in half and then one of the eighth pieces in half. **O**
- Raise the question, "What fraction do these two pieces represent?" holding or pointing to the one-fourth piece and one-eighth piece. **E, R**
- Have students label each piece with the appropriate fraction, then ask them to write down the mathematical equation they have created with all of their pieces (ie. $1/2 + 1/4 + 1/8 + 1/8 = 1$). **W, O**
- Discuss the hypothesis that students previously wrote down and if they still believe that their hypothesis is correct. **R**
- Draw a 2-D diagram of the process students have completed by cutting the blank sheet. Then, ask students to continue the diagram to the smallest size possible. Remind students that they can use the model in front of them to help them continue the pattern. **E, E-2**
- Raise the question, "From your model and diagram, what will $1/2 + 1/4 + 1/8 + 1/16 + 1/32 + 1/64 + \dots$ equal?" Students should be able to assume that they will get really close to one. **R**
- With the help of student diagrams and models, explain to students how one is the correct answer. **W, T**
- Ask students to write this series as an equation, which can be used to figure out each term of the series. **E**
- Introduce students to the symbol " Σ " and how it can be used to express an infinite series. **W, T**
- Raise the question, "What real life scenario can you think of where you will need to solve an infinite series?" **E-2**
- Discuss different occasions when professionals use infinite series. **E-2, R**
- Ask students to briefly write in their notebooks a strategy they could use to solve a different infinite series, such as $1/3 + 1/9 + 1/27 + 1/81$. **E-2, T, R**
- Discuss students' strategies and ideas on how to approach a different infinite series. Finally, ask students to solve this infinite series as well as two other infinite series for homework and to explain in at least one paragraph the strategies they used to reach their solution. Remind students to use appropriate mathematical notation. **R, E-2, T**

Additional Considerations:

- Students should be exposed to the concept of an infinite series and the various symbols used to represent the sum of a series, such as Σ . They will also be introduced to the concept of limits and infinity.
- Students who are having a greater degree of difficulty understanding the concepts during the simulation phase (or when students are using the 3-D model) will be paired with those who are grasping the concepts well.